Low-Complexity Hop Timing Synchronization in Frequency Hopping Systems

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Abstract—This paper considers the problem of synchronization in frequency hopping systems. A simple digital signal processing block along with the existing RF circuits forms a phase detector that generates a signal corresponding to the phase difference between the hop timings of the local and received signals. The two hop-timing acquisition algorithms proposed in this paper - based on energy detection - utilize the phase detector and result low-complexity implementations. In the first one, a simple cost function relying on the power sum definition estimates the hop timing. This estimator allows for only a limited acquisition range. Based on this study, we propose a hop-timing estimator with a full acquisition range which still maintains low-complexity. Furthermore, this paper analyzes the optimal threshold to detect the signal existence based on the maximum likelihood detection rule.

I. INTRODUCTION

Frequency hopping (FH) systems have got much attention due to the increasing demand for reliable and high security communications over wireless channels [1]. In FH systems, the available channel bandwidth, $W$, is subdivided into a number of non-overlapping frequency slots. In any signaling interval, the transmitted signal occupies one of the available frequency slots. The frequency slot in each signal interval is selected according to the applications. This FH system can gain anti-jamming capability through avoiding interference [2], [3].

A bulk of research in FH systems has recently focused on the optimal and suboptimal receivers for detecting the presence of FH signals [4]. Most of these works assume perfect synchronization between transmitter (base station) and receiver (local terminal), i.e., the exact time instant at which the signal switches in frequency and the set of candidate hopping frequencies are assumed to be known [4], [8]. Since pragmatic wireless links entail channel-induced interference, as well as timing errors, it is necessary to account for these effects when designing FH systems [5], [6]. One of the important issues in FH synchronization is to find out the start of a frequency hop, also known as the hop-timing estimation problem. Although we know the time-frequency code (TFC) at the local terminal, the misalignment between the incoming signal and the locally generated signal, referred as hop timing error, can result in considerable performance degradation [7], [9], [10].

In this paper, we propose two types of hop-timing estimation algorithms when assuming perfect frequency synchronization (i.e., the TFC is known), that allow for low-complexity implementations. The proposed estimators utilize a phase detector composed of the exiting RF circuits and a digital processing block.

There have been several works dealing with the hop-timing estimation which require complex implementations [2], [7]. Before deriving estimators, we introduce definitions regarding the power sum from the received samples. Then, we propose a simple cost function that yields the minimum at zero hop-timing error. Its implementation is simple, but the acquisition range of the hop-timing error is limited to $[-T_{\text{HOP}}/2, T_{\text{HOP}}/2]$, where $T_{\text{HOP}}$ is the hop duration (epoch). A hop-timing estimator with a full acquisition range of $[-T_{\text{HOP}}, T_{\text{HOP}}]$ is also possible using a power indicator introduced in this paper while maintaining low complexity. For a high performance operation of this estimator, it is crucial to find the optimal threshold for the power indicator, and an analysis for an optimal threshold based on the maximum likelihood (ML) rule is presented. Since the proposed estimator utilizes the RF circuits like mixers and data converters existing in wireless communication systems, the implementation requires the addition of only a simple digital signal processing block. Thus, a low-power low-cost design is viable.
II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a frequency hopping system, which hops over $N_H$ frequency bands. Figure 1 shows a proposed receiver structure with the phase detector and digital control block, where three frequencies are considered for example. The mixer and LPF generate output signal only when $r(t)$ and LO inputs have the same frequency. This combined with the data converter and simple digital processing unit forms a hybrid phase detector. We assume that frequency synchronization has been achieved so that the time-frequency code (TFC) is perfectly known, i.e., we do not consider the closed loop of the frequency hopping sequence of the base station. Figure 1 shows a proposed receiver structure with the phase detector and digital control block, and perfectly known. The signal part $s(t; f_k)$ is modeled as

$$
s(t; f_k) := \sqrt{E_s} e^{j \omega_k t},$$

where $\omega_k := 2\pi f_k$.

On the basis of frequency synchronization between the transmitter and receiver, the low-pass filtered signal of the mixer output is illustrated in Fig. 2 when there exists hop timing error $\Delta t$ defined as the time difference between the lock timing of the local terminal $t^L$ and that of the incoming signal $t^S$, i.e., $\Delta t := t^L - t^S$. In this example, a locally generated timing signal leads the received signal, which some part of the detected signal, which corresponds to $\Delta t$, will be lost as shown in Fig. 2.

In general, the local hop timing clock can lead or lag to the frequency hopping sequence of the base station. Figure 3 depicts four possible cases of the detected signal: 1) small lead, 2) large lead, 3) small lag, and 4) large lag. Relying on the detected signal, the objective of the hop timing synchronization in this paper is to find $\Delta t$ and adjust it so that the full energy detection during $T_{HOP}$ can be done.

In this paper, we propose two algorithms to efficiently estimate $\Delta t$ at low-complexity by processing the outputs of the analog-to-digital converter (ADC) in Fig. 2. One has a half acquisition range and the other achieves the full acquisition range.

III. HOP TIMING SYNCHRONIZATION

Denoting the hop duration as $T_{HOP}$, we assume that there are $N$ samples of the ADC outputs during $T_{HOP}$, where the ADC outputs are the samples of the detected signal. Then we can model the signal of the ADC output as

$$
x(i) = \begin{cases} 
    s(i) + n_0(i), & \text{when signal is detected} \\
    n_0(i), & \text{when signal is not detected}
\end{cases}$$

where $i = 0, 1, \ldots, N - 1$ and $n_0(i)$ is still zero-mean, white Gaussian noise. We note that due to perfect frequency synchronization, the signal $s(i)$ in (2) can be modelled as a constant with energy $E_s$, i.e., $s(i) = \sqrt{E_s}$. The sampled signal block of length $N$ is then represented as $x = \{x(0), x(1), \ldots, x(N - 1)\}$, based on which the estimators of the hop timing error are derived.

A. Estimator 1

For deriving the estimator, we first define two power sums with $x$, which are given by

$$P_{LEAD}(\Delta t) = \sum_{i=0}^{\lfloor N/2 \rfloor - 1} x^2(i),$$

$$P_{LAG}(\Delta t) = \sum_{i=\lfloor N/2 \rfloor}^{N-1} x^2(i),$$

where $P_{LEAD}(\Delta t)$ indicates the energy collected during the first half of $T_{HOP}$ and $P_{LAG}(\Delta t)$ during the last half of $T_{HOP}$. We note from (3) and Fig. 2 that $P_{LEAD}(0)$ and $P_{LAG}(0)$ should be equal if there is no hop timing error, i.e. $\Delta t = 0$. With this in mind, a simple estimator can be derived with the following cost function:

$$J_1(\Delta t) = \alpha \left( E[P_{LEAD}(\Delta t)] - E[P_{LAG}(\Delta t)] \right)^2,$$

where $\alpha$ is a normalization factor and $E[\cdot]$ means the expectation operator. Since (4) should be equal to zero for perfect synchronization, the hop timing error estimator will have the following form:

$$\hat{\Delta t}_1 = \min_{\Delta t \in [-T_{HOP}/2, T_{HOP}/2]} J_1(\Delta t).$$

Note that the evaluation of (5) is difficult for real systems because it requires full search. Instead of full search, we
introduce a simple solution of (5). Differentiating (4) with respect to \( \Delta t \) gives

\[
\frac{\partial J_1(\Delta t)}{\partial \Delta t} = 2\alpha (E[P_{\text{LEAD}}(\Delta t)] - E[P_{\text{LAG}}(\Delta t)]).
\]  

(6)

Figure 4 illustrates \( \frac{\partial J_1(\Delta t)}{\partial \Delta t} \) in (6), where the acquisition range is limited to the linear range of \([-T_{\text{HO}}/2, T_{\text{HO}}/2]\). Within the acquisition range, calculating \( (P_{\text{LEAD}}(\Delta t) - P_{\text{LAG}}(\Delta t)) \) can give the estimated hop-timing error \( \Delta t \), as shown in Fig. 4.

B. Estimator 2

As described in the previous subsection, the estimator 1 does not show the full acquisition range. In addition, it requires relatively complex operations, e.g., calculation of \( P_{\text{LEAD}}(\Delta t) \) and \( P_{\text{LAG}}(\Delta t) \). Therefore, we propose another estimator which can achieve the full acquisition range of \([-T_{\text{HO}}, T_{\text{HO}}]\] at lower complexity.

Note that \( P_{\text{LEAD}}(\Delta t) \) and \( P_{\text{LAG}}(\Delta t) \) in (3) integrate the energy detected during the sampling process, which are affected by the time duration of the signal existence. Also noting that the final objective of hop-timing synchronization is to find the hop timing error \( \Delta t \), we do not need to know the exact energy detected during the observation period. Instead, the time duration of the signal existence is sufficient information to estimate \( \Delta t \). Then, let us define a sample sequence \( x_p := \{x_p(0), x_p(1), \ldots, x_p(N-1)\} \), whose element is a power indicator defined as:

\[
x_p(i) = \begin{cases} 
1, & x^2(i) \geq \xi_T \\
0, & x^2(i) < \xi_T,
\end{cases}
\]  

(7)

where \( \xi_T \) is a threshold for power detection decision. Based on (7), (3) can be redefined as:

\[
T_{\text{LEAD}}(\Delta t) = \sum_{i=0}^{\lfloor \frac{\Delta t}{2} \rfloor -1} x_p(i), \\
T_{\text{LAG}}(\Delta t) = \sum_{i=\lfloor \frac{\Delta t}{2} \rfloor}^{N-1} x_p(i).
\]  

(8)

In addition, we also define the following

\[
T_{\text{ALL}}(\Delta t) = \sum_{i=0}^{N-1} x_p(i), \\
\bar{T}_{\text{ALL}}(\Delta t) = N - T_{\text{ALL}}(\Delta t).
\]  

(9)

Based on (9), the hop timing error can be estimated as

\[
\Delta \hat{t} = \begin{cases} 
T_{\text{ALL}}(\Delta t), & T_{\text{LEAD}}(\Delta t) \geq T_{\text{LAG}}(\Delta t) \\
-T_{\text{ALL}}(\Delta t), & T_{\text{LEAD}}(\Delta t) < T_{\text{LAG}}(\Delta t)
\end{cases}
\]  

(10)

which is also shown in Fig. 5, where \( T_s = T_{\text{HO}}/N \) is the sample duration. We note that the acquisition accuracy of (10) is the order of \( T_s \).

As described Fig. 5, the estimator 2 achieves full acquisition range of \([-T_{\text{HO}}, T_{\text{HO}}]\), while the estimator 1 in Fig. 4 does not. We also note that the estimator 2 requires only one comparator and three counters for (8) and (10), resulting in digitally-operated estimator with low complexity.

Now, let us consider the threshold \( \xi_T \) for power detection as in (7). Since the threshold \( \xi_T \) affects the estimator (10) in performance, it is required to derive the optimal threshold for power detection in (7).

For the noise sample \( x(k) = n_0(k) \), the distribution of \( x^2(k) \) can be obtained if \( n_0(k) \) is assumed to be AWGN, i.e., the square of a zero mean Gaussian random variable follows the central Chi-square distribution [3]:

\[
f_c(y) = \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp \left( -\frac{y}{2\sigma_n^2} \right),
\]  

(11)

where \( \sigma_n^2 \) is the noise power and \( y = n_0^2 \). For distribution of the signal sample, the square of a non-zero mean Gaussian variable becomes the non-central chi-square distribution [3]:

\[
f_{nc}(y) = \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp \left( -\frac{y + m_s}{2\sigma_n^2} \right) \cosh \left( \sqrt{\frac{y m_s}{\sigma_n^2}} \right),
\]  

(12)

where \( y = (\sqrt{E_s} + n_0)^2 \) and \( m_s = \sqrt{E_s} \). Then, the maximum likelihood (ML) detection rule for finding the threshold can be described as

\[
f_{nc}(y|y = n_0^2) \geq 1.
\]  

(13)

from which, we obtain the threshold \( \xi_T \) as

\[
\xi_T = \frac{\sigma_n^4}{m_s^2} \ln \left( \exp \left( \frac{m_s^2}{2\sigma_n^2} \right) + \sqrt{\exp \left( \frac{m_s^2}{\sigma_n^2} \right) - 1} \right).
\]  

(14)
It is observed from (14) that the threshold $\xi_T$ depends on the signal-to-noise ratio (SNR) defined as $E_s/\sigma_n^2$. Figure 6 shows $\xi_T$ corresponding to SNRs, which is found by (14). It is shown that as the SNR increases, the threshold decreases. It is intuitively understandable since at high SNRs the probability that the signal is detected is high, which coincides with low threshold.

IV. SIMULATIONS

In order to investigate the performance of our proposed estimators, we conducted simulations. The parameters in this simulation are following: the hop duration is $T_{HOP} = 300$ µsec, the sampling duration is $T_s = 1.89 \times 10^{-9}$ (equivalent to the sampling rate $F_s = 528$ MHz), and the number of samples is $N = T_{HOP} \cdot F_s = 158400$. We note that these parameters are considered in the multi-band orthogonal frequency division multiplexing (MB-OFDM) candidate in IEEE 802.15.3a standard, where the frequency band from 3.1 GHz to 4.8 GHz is used for three sub-bands of 528 MHz [8]. As a performance measure, we introduce the normalized mean square error (MSE) defined as

$$\text{MSE} := \frac{E[|\Delta t - \Delta \hat{t}|^2]}{E[|\Delta t|^2]},$$

(15)

Figure 7 compares the performance of two proposed estimators in terms of the normalized MSE of timing error. The hop-timing error for simulation is randomly generated within $[-T_{HOP}/2, T_{HOP}/2]$ which is the acquisition range of the estimator 1. However, we note that the full range of $[-T_{HOP}, T_{HOP}]$ can be tested for the estimator 2. It is observed that the estimator 2 shows good performance at high SNRs, while the estimator 1 gives good performance at low SNRs. We also note that the estimator equipped with the optimal threshold in (14) achieves better performance than that with the fixed threshold 0.5.

V. CONCLUSION

In this paper, we have derived two hop-timing estimators in FH systems relying on a phase detector that is composed of RF circuits like mixers data converters and a simple digital signal processing block. By using the power sum definition, we first proposed the cost function that has minimum at zero hop-timing error. Although this estimator has low complexity, it has a limited acquisition range. To achieve a full acquisition range while maintaining low complexity, another type of hop-timing estimator using power indicator was presented, where the optimal threshold to detect the signal existence was also derived. Simulation results demonstrated that the second estimator with the optimal threshold derived based on ML rule achieves good performance.

REFERENCES