Blind Iterative Channel Estimation and LDPC Decoding for OFDM Systems

Mi-Kyung Oh, Yeong-Hyeon Kwon, Jung-Hyun Park, and Dong-Jo Park
Dept. of EECS, KAIST; 373-1 Guseong-dong, Yuseong-gu, Daejon 305-701, Republic of Korea
{ohmik,rainangel,junghyunpark}@kaist.ac.kr, djpark@ee.kaist.ac.kr
Tel: +82-42-869-8038, Fax: +82-42-869-8038

Abstract—This paper aims at designing a novel blind iterative receiver for LDPC coded OFDM systems, where pilot symbols for estimating the channel are not inserted in the transmitter. Incorporating the differential QPSK scheme and LDPC decoder, we show that the decision-directed channel estimator can produce the channel information to improve performance of the LDPC decoder. The resulting blind iterative receiver can resolve the drawbacks of differential detection, an improved channel estimate and BER performance in the frequency selective channel, while keeping bandwidth efficiency. Simulation results verify performance of our scheme.

keywords: Low-density parity-check (LDPC), OFDM, channel estimation, DQPSK, blind iterative receiver.

I. INTRODUCTION

Although turbo codes have been shown to have near-capacity performance, they have a very high complexity. In [1], Gallager introduced a class of linear codes, known as low-density parity check (LDPC) codes. Later, in [2], it was rediscovered. Moreover recent design improvements and hardware realizations of LDPC codes have provided coding systems whose performance matches or even outperforms that of turbo codes while requiring lower complexity.

On the other hand, orthogonal frequency division multiplexing (OFDM) has been widely adopted by many standards, because it offers high data-rates at low decoding complexity [6]. Many OFDM schemes with various forward error correction algorithms, e.g., turbo codes, convolutional codes, and LDPC codes, have been pursued to mitigate a channel fading effect. Since the channel estimation in coded systems is essential for coherent demodulation and detection, most literatures have focused on obtaining the channel state information (CSI) using pilot symbols, resulting in bandwidth consumption [6], [7]. Thus the differential scheme (e.g., DQPSK) that does not require the CSI has got much attention due to its spectral-efficiency. However, it loses 3 dB performance compared with coherent detection scheme.

In this paper, we propose to use previously made decisions from the LDPC decoder output instead of pilot symbols for estimating the channel. Thus we have derived a decision-directed channel estimator that can be used in the LDPC decoder. Moreover, we show that although the differential scheme (DQPSK) is used in our system, performance gain can be obtained by employing the fact that there is a significant performance advantage to use the CSI for the LDPC decoder.

In our LDPC-coded OFDM systems combined with DQPSK scheme, the CSI is estimated with the output of the LDPC decoder. The estimated CSI can then be incorporated to coherently perform LDPC decoding, which results in performance improvement while keeping bandwidth efficiency. In this fashion, the blind iterative LDPC decoding and channel estimation are performed.

II. SYSTEM MODEL

Notation: Upper (lower) bold face letters denote matrices (column vectors). Superscript $(\cdot)^T$ indicates Hermitian, $(\cdot)^H$ transpose. $	ext{diag}[\mathbf{x}]$ stands for a diagonal matrix with $\mathbf{x}$ on its main diagonal. Matrix $\mathbf{D}_N(\mathbf{h})$ with a vector argument denotes an $N \times N$ diagonal matrix $\mathbf{D}_N(\mathbf{h}) = \text{diag}[\mathbf{h}]$. $[\mathbf{A}]_{k,m}$ stands for the $(k,m)$th entry of a matrix $\mathbf{A}$; $x[m]$ for the $m$th entry of the column vector $\mathbf{x}$; and $[\mathbf{F}_N]_{m,n} = N^{-1/2}\exp(-j2\pi mn/N)$ for the $N \times N$ FFT matrix.

A. Transmitter and Channel

We consider the discrete-time equivalent baseband model of an LDPC-coded OFDM system communicating over frequency-selective channels, shown in Fig. 1. The information data $\mathbf{b}$ with length $N_b$ is encoded into $\mathbf{c}$ of length $2N$ by the systematic LDPC encoder $G_{sys}$, where the code rate is $R := N_b/2N$. The encoded $\mathbf{c}$ is modulated by DQPSK modulator to form $\mathbf{e}$ with length $N$. In the DQPSK modulator, pairs of LDPC encoded bits are used to form a sequence of complex differential symbol, where the four symbols intended for communications are mapped to the following set: $\Phi = \{0, \pi/2, \pi, -\pi/2\}$.

Following the DQPSK modulation, we take OFDM operation. Specifically, we implement $N$-point inverse FFT (IFFT) on $\mathbf{e}$ and insert the cyclic prefix (CP) to form $\mathbf{x}$ of length $P := N + L$. After parallel to serial (P/S) conversion, each data segment is transmitted through the multipath channel.

The frequency-selective channel in discrete-time baseband equivalent form is denoted by $h := [h(0), \ldots, h(L)]^T$ with
order $L$. This channel incorporates transmitter (receiver)-filter $g_{tx}(t)$ ($g_{rx}(t)$) and the frequency-selective multipath $g(t)$: i.e., $h(l) = (g_{tx}(t) * g(t) * g_{rx}(t))|_{l=IT}$, where $*$ denotes convolution, and $T$ is the sampling period. Then the samples of the receive-antenna filter output can be written as

$$\bar{y}(n) = \sum_{l=0}^{L} h(l)x(n-l) + \bar{w}(n),$$

where $\bar{w}(n)$ is zero-mean, white Gaussian noise with variance $\sigma_w^2 := N_0/2$. The sequence $\bar{y}(n)$ is then serial to parallel (S/P) converted into $\bar{y}$ of length $P$ as in Fig. 2.

**B. Receiver**

After removing the CP and OFDM demodulation on $\bar{y}$, we can obtain the following input-output relationship:

$$y = D_N(\bar{h})c + w,$$

where $D_N(\bar{h})$ is a diagonal matrix, and its element is the channel frequency response values on the FFT grid, i.e., $\bar{h} := [h(0), \ldots, h(2\pi(N - 1)/N)]^T$, with $h(2\pi n/N) := \sum_{l=0}^{L} h(l) \exp(-j2\pi ln/N)$. We note that (2) renders a set of flat-fading sub channels equivalent to the frequency-selective channel.

We now perform DQPSK demodulation on (2) to detect $\hat{c}$, which is fed into the LDPC decoder. For most algorithms that employ the differential scheme and error correction codes, detection process of the transmitted symbols ends here. However, this is not the case with our approach. Since there is a significant performance advantage to use the CSI for coded DQPSK system, we estimate the channel afterwards. We notice that we do not insert pilot symbols at the transmitter. Instead, we use the decision values, output of the LDPC decoder, to estimate the channel, which is so called “decision-directed channel estimation”.

In this way, LDPC decoding and channel estimation are performed iteratively without pilot symbols in our blind iterative receiver. At the last iteration, we obtain the transmitted information bits $\hat{b}$ from the LDPC decoder output $s$, which is easy since we exploit the systematic LDPC encoder $G_{sys}$.

**III. BLIND ITERATIVE RECEIVER**

In this section, we propose the technique of improving BER performance of LDPC coded differential OFDM systems by using the CSI that is obtained from the decision-directed channel estimator. Before we specify the proposed algorithm, we first start with LDPC decoding algorithm.

**A. LDPC Decoding**

Once the DQPSK demodulated bits are obtained, the log-likelihood ratios (LLRs) for the LDPC decoder are computed. It is convenient to specify a code rate $N_b/2N$ LDPC code with an $(2N - N_b) \times 2N$ parity check matrix $H$ using the Tanner graph representation. The Tanner graph is a bipartite graph, where the nodes on the bottom are associated with the bit nodes, and the nodes on the top are associated with the check nodes, shown in Fig. 3.

The sum-product algorithm (SPA), also known as a message-passing algorithm, is used to decode the LDPC codes [3]. To describe the SPA, we first introduce the notations. Letting that $j$ is the column weight and $k$ is the row weight in $H$, $Q_1(m, t)$ denotes the set of check nodes that are connected to the bit node $t$ where $t = 1, \ldots, 2N$ and $m = 1, \ldots, j$. $Q_2(m, l)$ indicates the set of bit nodes that participates in the $l$th parity-check equation, i.e., the positions of “1”s in the $l$th row of the parity check matrix, where the check node index $l = 1, \ldots, 2N - N_b$ and $m = 1, \ldots, k$. The LLRs are defined as: $LR(q_{l,t}) := \log(q_{l,t}^+/q_{l,t}^-)$, where $q_{l,t}^+$ denotes the probability that the $t$th bit node has the value $x$, given the information obtained via the check nodes other than check node $l$, and $LR(r_{l,m}) := \log(r_{l,m}^+/r_{l,m}^-)$, where $r_{l,m}^+$ denotes the probability that a check node $l$ is satisfied when bit $t$ is fixed to a value $x$ and the other bits are independent with probabilities $q_{t',l}^-$ with $t' \neq t$. The procedure of the SPA is then summarized as [3], [4]:

**Initialization**

Each bit node $t$ is assigned to an a priori LLR $LR(f_t)$.

$$LR(f_t) = (4\mathbb{E}_s/N_o)y_t,$$

$$LR(r_{l,t}^{(0)}) = 0, t = 1, \ldots, 2N \text{ and } l = 1, \ldots, 2N - N_b,$$

where $\mathbb{E}_s/N_o$ is SNR and $y_t$ denotes the received symbol at the $t$th node.
Iteration

The bit-to-check messages and check-to-bit messages in the \( \nu \)th LDPC decoding iteration are obtained as:

**Bit-to-Check messages:** \( LR(q_{1,Q1(1,m)}^{(\nu)}) \)

\[
LR(q_{1,Q1(1,m)}^{(\nu)}) = LR(f_t) + \sum_{m' \neq m} LR(r_{(t,Q1(1,m'))}^{(\nu-1)}),
\]

\( t = 1, \ldots, 2N; \ m = 1, \ldots, j. \)

**Check-to-Bit messages:** \( LR(r_{Q2(m,l)}^{(\nu)}) \)

\[
LR(r_{Q2(m,l)}^{(\nu)}) = g^{-1} \left[ \prod_{m' \neq m} g(LR(q_{Q2(m',l)}^{(\nu)})) \right],
\]

\( l = 1, \ldots, 2N - N_h; \ m = 1, \ldots, k. \)

\( g(x) := \tanh(-x/2). \)

Output

The *a posteriori* LLR for a bit node \( t \), defined as \( LR(p_t^{(\nu)}) \) in the \( \nu \)th LDPC decoding iteration, is calculated by gathering all the extrinsic information from check nodes that connect to it, which is given by

\[
LR(p_t^{(\nu)}) = LR(f_t) + \sum_{s=1}^{j} LR(r_{Q1(1,m)}^{(\nu)}).
\]

**Decision**

if \( LR(p_t^{(\nu)}) > 0, \ s(t) = 1 \), else \( s(t) = 0. \)

**Termination**

if the syndrom \( Hs = 0 \), decoding stops, else, return to *Iteration*,

where \( \nu = 1, \ldots, \nu_{\text{max}} \) with \( \nu_{\text{max}} \) representing the predefined maximum number of LDPC decoding iterations. We note that the iteration index \( \nu \) denotes LDPC decoding iteration itself.

**B. Initial LDPC Decoding and Decision-Direct Channel Estimation**

For the first iteration in our receiver, the CSI is not known for the LDPC decoder so that hard decision values of the DQPSK demodulator output are used for the LDPC decoder. It may give the poor performance in LDPC decoding. Thus we take the following scheme for overcoming this poor performance: we put decision-directed channel estimator following the LDPC decoder in order to obtain the CSI and the LDPC decoder and channel estimator are iteratively performed. Specifically, the hard decision values of the bit nodes at the LDPC decoder output are considered as pilot symbols for estimating the CSI. With these decision values, the maximum likelihood (ML) channel estimator is considered.

Letting us define \( 2N \) decision values as \( s^{(1)} := [s^{(1)}(0), \ldots, s^{(1)}(2N - 1)]^T \) for the first iteration, we have the following input-output relationship

\[
y = B^{(1)}h + w,
\]

where \( y := [y(0), \ldots, y(N - 1)]^T \) is the OFDM demodulated signals as in (2), and \( B^{(1)} := \sqrt{N} \cdot \text{diag}[s^{(1)}(0), \ldots, s^{(1)}(N - 1)] \). \( F \) with \( s^{(1)} \) denoting the DQPSK modulated symbols of \( s^{(1)} \) and \( F \) representing the first \( L + 1 \) columns of \( F_N \). From (9), the linear minimum mean-square error (LMMSE) channel estimator for the first iteration is given by [6]

\[
\hat{h}_{\text{LMMSE}}^{(1)} := (\sigma_n^2 R_h^{-1} + B^{(1)T}B^{(1)})^{-1}B^{(1)T}y,
\]

where \( R_h := E[hh^T] \) is the channel covariance matrix, and \( \sigma_n^2 \) denotes the noise variance.

Because \( R_h \) is unknown in practice, the maximum likelihood (ML) channel estimator can be used. The ML channel estimator takes the following form [6]

\[
\hat{h}_{\text{ML}}^{(1)} := (B^{(1)T}B^{(1)})^{-1}B^{(1)T}y.
\]

Based on (11), the channel frequency response on FFT grid, i.e., the estimate of \( D_N(\hat{h}^{(1)}) \) corresponding to \( \hat{h}_{\text{ML}}^{(1)} \) can be computed as

\[
D_N(\hat{h}^{(1)}) = \sqrt{N} \cdot \text{diag}[F\hat{h}_{\text{ML}}^{(1)}].
\]

**C. Iterative LDPC decoding and Decision-Directed Channel Estimation**

From the second iteration, the LDPC decoder now incorporates the CSI, where the absolute value of the CSI estimate, obtained from (12), is properly scaled with the output of DQPSK demodulator.

Noticing that \( s^{(\mu)} \) is the DQPSK modulated symbols of the LDPC decoder output \( s^{(\mu)} \) in the \( \mu \)th iteration, we formulate the following

\[
y = B^{(\mu)}h + w,
\]

where \( B^{(\mu)} := \sqrt{N} \cdot \text{diag}[s^{(\mu)}] \cdot F \). Thus the ML estimator for the second iteration can be expressed as

\[
\hat{h}_{\text{ML}}^{(\mu)} := (B^{(\mu)T}B^{(\mu)})^{-1}B^{(\mu)T}y.
\]

In this way, the \( \mu \)th CSI also can be obtained by using the LDPC decoder output in the \( \mu \)th iteration, where \( \mu = 1, 2, \ldots, \mu_{\text{max}} \) with \( \mu_{\text{max}} \) denoting the predefined maximum number of iterations in our iterative receiver. It is observed that performance of the estimator in (14) depends on the LDPC decoder output which is also affected by the previous channel estimator. We note that the iterative channel estimator can give more accurate CSI, which in turn enhances performance of the LDPC decoder.
D. Summary and Discussion

We have derived the blind iterative LDPC decoding and channel estimation for differential OFDM systems. In the following, we summarize the proposed algorithm in these steps:

**Step 1** Initialization: From the DQPSK demodulator output, the initial LDPC decoding and channel estimation are performed.

**Step 2** LDPC decoder: Combined with the CSI estimated from the previous iteration, LDPC decoding is carried out.

**Step 3** Iterative channel estimation: Based on the hard decisions of the LDPC decoder outputs, the channel estimation is refined.

**Step 4** Stop: The iterative algorithm stops if the number of iterations exceeds a predetermined value. Otherwise, return to Step 2.

The major advantages of our iterative algorithm are listed:

- As opposed to the existing algorithms [7], [9], our algorithm shows high spectral efficiency: because of the advantage of the DQPSK scheme and LDPC decoding, we do not insert pilot symbols for estimating the frequency-selective channel.
- The drawbacks with differential detection are resolved by exploiting the CSI and advantages of iterative receiver.

IV. SIMULATION RESULTS

To confirm performance of our blind iterative receiver, we conducted simulations. For the simulations, the HyperLAN/2 channel model B is used to generate the frequency selective channels of order $L = 15$. The parameters regarding the LDPC codes for the proposed system are detailed in Table I. In addition, we use DQPSK modulation, pairs of permuted encoded-bits are used to form a sequence of complex symbol, so that the OFDM block length (FFT size) is $N = 1024$.

We plot BER versus SNR in Fig. 4. The ideal case corresponding to a perfectly known CSI is also depicted as a benchmark, for which there is about 3 dB gap at $BER = 10^{-2}$ between the case without CSI and the case with perfect CSI. The results in Fig. 4 show BER performance improvement with the increased iterations and incorporate our claim that the CSI estimate can help LDPC decoding, which results in better BER performance. It is also observed that for $\mu = 2$, BER dramatically decreases. However, the improvement is relatively small for $\mu > 2$. We also note that using well-designed LDPC codes will enhance performance of channel estimator by the reliable estimates of data symbols from the LDPC decoder, which in turn improves BER performance.

![BER performance in HyperLAN2 channel.](image)

Fig. 4. BER performance in HyperLAN2 channel.

V. CONCLUSIONS

We have investigated blind iterative LDPC decoding and channel estimation for OFDM systems. We employed previously made decisions from the LDPC decoder output instead of pilot symbols to estimate the channel so that performance of the differential scheme combined with LDPC codes can be enhanced. Simulation results verified our claim that BER performance is improved. The future work will be the efficient use of the CSI estimate in coherent detection of DQPSK scheme to further enhance BER performance.

ACKNOWLEDGEMENT

This work was supported in part by the Ministry of Information & Communications, Korea, under the Information Technology Research Center (ITRC) Support Program.

REFERENCES


![BER performance in HyperLAN2 channel.](image)

**TABLE I**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{sys}$</td>
<td>$(1024,2048)$</td>
</tr>
<tr>
<td>$H$</td>
<td>$(1024,2048)$</td>
</tr>
<tr>
<td>Column weight</td>
<td>3</td>
</tr>
<tr>
<td>Code rate</td>
<td>0.5</td>
</tr>
</tbody>
</table>

![BER performance in HyperLAN2 channel.](image)