Performance Analysis of UWB Multiple Access Using Multi-code Based PPM

Sung-Yoon Jung *, Dong-Jo Park
Department of Electrical Engineering and Computer Science
Korea Advanced Institute of Science and Technology (KAIST)
373-1 Guseong-dong, Yuseong-gu, Daejeon, Republic of Korea 305-701
E-mail: syjung@kaist.ac.kr, djpark@ee.kaist.ac.kr

Abstract—In this paper, we propose an UWB time-hopping multiple-access (TH-MA) scheme using multi-code based PPM. The multi-code signal, which is generated by spreading the desired user's M transmitting bits with M orthogonal codes, is pulse-position modulated. At the receiver, the received signal is demodulated through two stages, pulse-position demodulation stage and de-spreading stage. In the first stage, the multi-coded PPM signal is demodulated through the Bayes decision rule. Then, in the second stage, the demodulated multi-code signal is de-spreaded by using the M orthogonal spreading codes that are used in the transmitter. The BER performance and bit transmission rate (R_b) of the proposed scheme is analyzed. Numerical results show that the proposed scheme has a much higher bit transmission rate than the conventional PPM scheme as M increases and also has better BER performance than the conventional PPM scheme in the same bit transmission case because it can achieve an additional spreading gain.

I. INTRODUCTION

The Ultra-Wide Bandwidth (UWB) technology has been recently considered as a viable solution for high-speed indoor short range wireless communication systems such as Wireless Personal Area Networks (W-PAN), because of its robustness to severe multipath conditions and low cost and low power implementation.

The UWB system introduced in [1] shows considerable abilities in indoors multiple access (MA) radio communications. In this system, data is transmitted using extremely short pulses with duration less than 1ns. This technique is called as impulse radio (IR) and since the transmitted pulses are extremely short, the bandwidth of this system is a few hundred times larger than the bandwidth of other systems for the same applications [2]. The modulation for this system is binary pulse position modulation (PPM), which is used along with TH-CDMA for a multiple access application. It has been shown that in an AWGN, the system has capacity to support a relatively very high total transmission rate using the single user correlator receiver. In [3], the the M-ary PPM technique, which is the extension of the system described in [1] [2], is considered in order to increase the transmission rate and Signal-to-Noise Ratio (SNR) in the receiver end.

In this paper, we propose an UWB time-hopping multiple-access (TH-MA) scheme using multi-code based PPM. The multi-code signal, which is generated by spreading the desired user’s M transmitting bits with M orthogonal codes, is pulse-position modulated. At the receiver, the received signal is demodulated through two stages, pulse-position demodulation stage and de-spreading stage. In the first stage, the multi-coded PPM signal is demodulated through the Bayes decision rule. Then, in the second stage, the demodulated multi-code signal is de-spreaded by using the M orthogonal spreading codes that are used in the transmitter. The BER performance and bit transmission rate of the proposed scheme is analyzed. As a result, we can see that the proposed scheme can achieve very high transmission rate and due to this merit of the very high transmission rate, it can achieve benefits in enhancing the BER performance by controlling the length of the spreading gain. Through numerical results, the performance of the proposed scheme is guaranteed and compared with other schemes.

II. SIGNAL MODEL

The νth user’s transmitting signal conveying information exclusively in the time shifts is

\[ x^{(\nu)}(t) = \sum_{k=0}^{\infty} w(t - kT_f - c^{(\nu)}_k T_c - m^{(\nu)}_{d^{(\nu)}_{(k/N_s)}(k)} T_m) \]  \hspace{1cm} (1)

where \( w(t) \) represents the pulse waveform with pulse width \( T_w \). The parameter \( k \) means a frame index (1 pulse/frame) and \( T_f \) is frame time duration. The \( c^{(\nu)}_k \) is a time-hopping sequence assigned to user \( \nu \). It is periodic with a period \( N_s \) and each sequence element is an integer in the range of \( 0 \leq c^{(\nu)}_k \leq N_s \). \( T_c \) is a time-hopping shift parameter. The maximum time-hopping duration, \( N_s T_c \), is less than \( T_f \). \( m^{(\nu)}_{d^{(\nu)}_{(k/N_s)}(k)} \) is a pulse-position modulation parameter depending on \( d^{(\nu)}_{(k/N_s)}(k) \), and \( T_m \) is a modulation time shift parameter.

The signal \( d^{(\nu)}_{(k/N_s)}(k) \), which we call it as a multi-code signal, is generated by spreading user \( \nu \)'s M bits with M orthogonal codes of the spreading gain \( N_s \).

\[ d^{(\nu)}_{(k/N_s)} = [d^{(\nu)}_{(k/N_s)}(0), \ldots, d^{(\nu)}_{(k/N_s)}(N_s - 1)]^T \]

where

\[ b^{(\nu)}_{(k/N_s)} = [b^{(\nu)}_1(k/N_s), \ldots, b^{(\nu)}_M(k/N_s)]^T \]

\[ S^{(\nu)} = [s^{(\nu)}_1, \ldots, s^{(\nu)}_M] \]

\[ s^{(\nu)}_m = [s^{(\nu)}_m(0), \ldots, s^{(\nu)}_m(N_s - 1)] \]  \hspace{1cm} (3)
are the $M$ bit vector, spreading code matrix and $m$th spreading
code of user $\nu$, respectively. Each bit, $\nu_q([k/N_s])$ $(m = 1, \ldots, M)$ is random binary with zero mean and unit variance.
We let $M$ be an odd number for the multi-code signal $d_{[k/N_s]}^{(\nu)}(k)$ not to have zero value. Then $d_{[k/N_s]}^{(\nu)}(k)$ has $M+1$
levels with values $\{-M, -M+2, \ldots, 1, 1, \ldots, M-2, M\}$.

From $d_{[k/N_s]}^{(\nu)}(k)$, the pulse position modulation parameter, $m_{d_{[k/N_s]}^{(\nu)}(k)}$ is given as shown below:

$$m_{d_{[k/N_s]}^{(\nu)}(k)} = \frac{d_{[k/N_s]}^{(\nu)}(k) + M}{2}.$$  \hspace{1cm} (4)

III. RECEIVER DESIGN

When $N_u$ users are active in the multiple-access system, the received signal $r(t)$ can be modelled as

$$r(t) = \sum_{\nu=1}^{N_u} A^{(\nu)} x^{(\nu)}(t - \tau^{(\nu)}) + n(t) \hspace{1cm} (5)$$

where $A^{(\nu)}$ and $\tau^{(\nu)}$ are the received signal amplitude and delay of user $\nu$, respectively and $n(t)$ is AWGN. If the receiver
wants to demodulate user one’s first $M$ transmitted bits, then, Eq. (5) can be rewritten as

$$r(t) = A^{(1)} x^{(1)}(t - \tau^{(1)}) + n_{tot}(t), \hspace{1cm} t \in T$$  \hspace{1cm} (6)

where $T = [\tau^{(1)}, N_uT + \tau^{(1)}]$ and $n_{tot}(\cdot)$ includes AWGN and other users’ received signals.

A. Multi-code based PPM Demodulation Using Bayes Decision Rule

In the conventional PPM signal [3], it is usually assumed that the probability of the pulse position occurrence is uniformly distributed.
However, contrary to the conventional one, the each pulse position occurrence probability of the multi-code based PPM signal is different. Therefore, in the receiver end, we use the Bayes decision rule because it is well known that when a priori probability is different, the decision rule that minimizes the probability of error is the Bayes decision; i.e. choose the hypothesis with the largest a posteriori probability [4].

Let $p_j = P(m_{d_{[k/N_s]}^{(\nu)}(k)} = j)$ denote a priori probability that the multi-code based PPM signal, $m_{d_{[k/N_s]}^{(\nu)}(k)}$ has position $j$ ($j = 0, \cdots, M$). If we assume a random binary spreading code, it is easily proven that $p_j$ follows the binomial distribution:

$$p_j = P(m_{d_{[k/N_s]}^{(\nu)}(k)} = j) = MC_j \left( \frac{1}{2} \right)^M$$  \hspace{1cm} (7)

where $MC_j$ is defined as:

$$MC_j = \binom{M}{j} = \frac{M!}{j!(M-j)!}.$$  \hspace{1cm} (8)

After the received signal is passing through the $M+1$ template pulses for each frame ($k = 0, \cdots, N_s-1$), followed by the sampler, the decision variables are obtained as follows:

$$t_{k,j} = \int_{t \in T_k} r(t) \cdot \nu_{d_{k,j}}(t - \nu t_k) - \nu_{e_k} \nu^{(1)} \nu t \hspace{1cm} (9)$$

with $T_k = [\nu t_k + \nu^{(1)} \nu, (k+1)T + \nu^{(1)} \nu]$ and $\nu_{d_{k,j}} = \nu(t - jT_m)$ for $j = 0, \cdots, M$.

Based on the Bayes decision rule and using the simplification process, the multi-code based PPM signal and multi-code signal can be demodulated as shown below:

$$\hat{m}_{d_{0}}^{(1)}(k) = \arg \max_j \left( \frac{P_{w}}{\sigma_n^2} t_{k,j} + \ln p_j \right)$$  \hspace{1cm} (10)

$$\hat{d}_{0}^{(1)}(k) = 2 \cdot \hat{m}_{d_{0}}^{(1)}(k) - M.$$  \hspace{1cm} (11)

where $\sigma_n^2$ is the variance of $n_{tot}(\cdot)$.

B. De-spreading of Multi-code Signal

After passing through the multi-code based PPM demodulation stage, user 1’s $M$ transmitted bits are demodulated by de-spreading the demodulated multi-code signal as follows:

$$\hat{z}_{0}^{(1)} = S^T \hat{d}_{0}^{(1)}$$  \hspace{1cm} (12)

$$\hat{d}_{0}^{(1)} = [\hat{\nu}_{0}^{(1)}(0), \cdots, \hat{\nu}_{M}^{(1)}(0)]^T$$

$$= \text{sign}(\hat{z}_{0}^{(1)})$$  \hspace{1cm} (13)

where $\hat{\nu}_{M}^{(1)} = [\hat{\nu}_{0}^{(1)}(0), \cdots, \hat{\nu}_{M}^{(1)}(N_s-1)]^T$.

IV. PERFORMANCE ANALYSIS

In this section, we consider the BER performance and the bit transmission rate ($R_b$) of the proposed scheme as a performance measure.

A. Error Probability Analysis of the Multi-code based PPM Demodulation

To evaluate the BER performance, we first consider the error probability of the multi-code based PPM demodulation $P_{mc/ppm}(e)$ for the $k$th frame ($k = 0, \cdots, N_s-1$). Because $P_{mc/ppm}(e)$ is same for each frame in the same symbol duration, we omit the index $k$. Although $P_{mc/ppm}(e)$ can theoretically be calculated, this computation is often impractical [5]. In such cases bounds on $P_{mc/ppm}(e)$ that are easy to calculate are desirable, and several bounds have been presented in the literature [7] for the two hypothesis decision problem. However, we have to deal with the multi-hypothesis problem. So, we use the union bound [8] to evaluate $P_{mc/ppm}(e)$.

A priori probability of the multi-code based PPM signal is defined in (7) and let $f_{j}(x) = f(x)m_{d_{[k/N_s]}^{(\nu)}(k)} = j$ be the conditional probability density. Then, the pairwise error probability is given as [6]:

$$P_{u,v}(e) = \int \min\{p_{u,v}(x), p_{u,v}(x)\} \hspace{1cm} \text{dx}$$

$$\leq \int\left[\min\{p_{u,v}(x)\}e^{[\min\{p_{u,v}(x)\}]\alpha} \hspace{1cm} \text{dx} \right.$$  \hspace{1cm} (14)

The latter inequality follows immediately from the fact that:

$$\min\{a, b\} \leq a^\alpha b^{1-\alpha} \hspace{1cm} \text{for} \hspace{1cm} a, b > 0 \hspace{1cm} \text{and} \hspace{1cm} 0 \leq \alpha \leq 1.$$  \hspace{1cm} (15)
Then, the union bound for $P_{mc/ppm}(e)$ in terms of $P_{u,v}(e)$ can be written as:

$$P_{mc/ppm}(e) \leq \sum_{u<v} P_{u,v}(e)$$

$$\leq \sum_{u<v} p_u^a p_v^{1-a} \int f_u(x)^a f_v(x)^{1-a}dx (16)$$

where

$$0 \leq \alpha \leq 1 \quad \text{all } u, v = 0, \cdots, M. \quad (17)$$

In particular, the union bound for $\alpha = \frac{1}{2}$, called as Bhattacharyya bound, is:

$$P_{mc/ppm}(e) \leq \sum_{u<v} \sqrt{p_u p_v} \int \sqrt{f_u(x) f_v(x)}dx. \quad (18)$$

To complete the derivation of $P_{mc/ppm}(e)$, we need to know the conditional probability density $f_j(x)$. By substituting (6) for (9), we can reformulate (9) as:

$$t_{k,j} = \begin{cases} A(1)^{1/2} \sqrt{E_w} + n_{k,j}, m_{d(k)/n_{j}}(k) = j, \\ n_{k,j}, \text{otherwise} \end{cases} \quad (19)$$

where

$$E_w = \int_{-\infty}^{\infty} w^2(\xi)d\xi \quad (20)$$

$$n_{k,j} = \int_{t \in T_k} n_{tot}(t) \cdot v_{k,j}^{(1)}(t - k T_T - e_{c(k)}^{(1)} T_T - \tau^{(1)}(t))dt \quad (21)$$

for $k = 0, \cdots, N_s - 1$ and $j = 0, \cdots, M$ and we assume that $n_{k,j}$ is independent Gaussian random variable which follows the distribution, $N(0, \sigma_n)$ for analytical simplicity. Therefore, the conditional probability density $f_j(x)$ can be derived from (19) as follows:

$$f_j(x) = \frac{1}{\sqrt{2\pi \sigma_n}} \exp \left[ -\frac{(x - A(1)^{1/2} \sqrt{E_w})^2}{2 \sigma_n^2} \right] \quad (22)$$

**B. BER Analysis of De-spreaded Multi-code Signal**

For the analytical convenience, we reformulate (11) as follows:

$$\hat{d}_0^{(1)}(k) = d_0^{(1)}(k) + e(k) \quad (23)$$

where $e(\cdot)$ denotes the multi-coded PPM demodulation error (the error value due to incorrect pulse position decision).

To derive BER, we have to know the distribution of $e(\cdot)$. However, $e(\cdot)$ does not follows some kinds of the well-known distribution at all because it is a specially defined error. Therefore, we have to derive the distribution of $e(\cdot)$.

From the fact that $\hat{d}_0^{(1)}(k)$ lies in the set:

$$D = \{-M, -M + 2, \cdots, -1, 1, \cdots, M - 2, M\}, \quad M : \text{odd number} \quad (24)$$

the possible $e(k)$ for each $\hat{d}_0^{(1)}(k) = m$ ($m \in D$) can be written as:

$$\begin{align*}
M & \Rightarrow e(k) \in \{0, -2, \cdots, -2M\} \\
M - 2 & \Rightarrow e(k) \in \{2, 0, \cdots, -2M + 2\} \\
& \vdots \\
-M + 2 & \Rightarrow e(k) \in \{M - 2, \cdots, -4, -2\} \\
-M & \Rightarrow e(k) \in \{M, \cdots, -2, 0\}
\end{align*} \quad (25)$$

If we assume that the possible $e(k)$ for each $\hat{d}_0^{(1)}(k) = m$ occurs with equal probability, the $P\{e(k) = n\}$ can be calculated as follows:

$$P\{e(k) = n\} = \begin{cases} \sum_{i=0}^{M} C_i \left(\frac{1}{2}\right)^{M} \cdot (1 - P_{mc/ppm}(e)) \\
\sum_{i=n/2}^{M} C_i \left(\frac{1}{2}\right)^{M} \frac{P_{mc/ppm}(e)}{M} \\
\text{for } n = 0 \\
\text{for } n = \pm 2, \cdots, \pm 2M - 2, \pm 2M 
\end{cases} \quad (26)$$

Due to the symmetry property, the mean of $e(k)$ is zero and the variance of $e(k)$, $\sigma^2_e$ can be calculated as:

$$\sigma^2_e = \left(\frac{1}{2}\right)^{M-3} \sum_{k=1}^{M} k^2 \left(\sum_{i=0}^{M-k} C_i\right) \frac{P_{mc/ppm}(e)}{M} \quad (27)$$

Now, we rewrite (12) as follows:

$$z_0^{(1)} = S^{(1)^T} d_0^{(1)} = S^{(1)^T} (d_0^{(1)} + e)$$

$$= N_s b_0^{(1)} + \tilde{e} \quad (28)$$

where $e = [e(0), \cdots, e(N_s - 1)]^T$ and

$$\tilde{e} = [\tilde{e}(1), \cdots, \tilde{e}(M)]^T = S^{(1)^T} e. \quad (29)$$

If we assume that $E\{\tilde{e}(i)\tilde{e}(j)\} = 0$, $i \neq j$ for $i,j = 1, \cdots, M$, the mean and variance of $\tilde{e}$ can be derived as follows:

$$E\{\tilde{e}\} = 0 \quad (30)$$

$$\text{Var}\{\tilde{e}\} = N_s \sigma^2_e \text{I}. \quad (31)$$

where $0$ is $M \times 1$ vector and $I$ is $M \times M$ identity matrix.

If $N_s$ is large enough ($N_s > 7$), the distribution of $\tilde{e}$ can be approximated as a Gaussian distribution, $N(0, \sqrt{N_s} \sigma_e I)$ due to the Central Limit Theorem [9]. Then, the BER performance of the proposed scheme is equivalent to calculating the BER performance of the following hypothesis:

$$r = \begin{cases} +N_s + \eta, \quad \text{when bit 1 was sent} \\
-N_s + \eta, \quad \text{when bit 0 was sent} 
\end{cases} \quad (32)$$

where $\eta$ is the error-noise with distribution $N(0, \sqrt{N_s} \sigma_e)$ and $\sigma_e$ is shown in (27).

Therefore, the BER performance of the proposed scheme can be derived as shown following:

$$\text{BER}_{\text{proposed}} = Q\left(\frac{\sqrt{N_s}}{\sigma_e}\right) \quad (33)$$
C. Bit Transmission Rate \((R_b)\)

Next, we consider the Bit Transmission Rate \((R_b)\) of the proposed scheme as another performance measure.

By letting \(T_w = T_m, T_s = (M + 1)T_m, T_f = N_h T_s\) and \(T_s = N_s T_f\), the bit transmission rate \(R_b\) of the proposed scheme can be derived as follows:

\[
R_{b,\text{proposed}} = \frac{M}{T_s} = \frac{M}{N_s N_h (M + 1) T_m} \text{ (bps)}. \tag{34}
\]

V. NUMERICAL RESULTS

In this section, we present some numerical results of the bit transmission rate and the BER performance of the proposed scheme. The bit transmission rate of the proposed scheme is shown in Fig. 1 when \(N_s = 32, N_h = 5\) and \(T_m = 0.156\ ns\). For comparison, the bit transmission rate of the PPM scheme [3] is included:

\[
R_{b,\text{ppm}} = \frac{M}{T_s} = \frac{M}{N_s N_h 2^M T_m} \text{ (bps)} \tag{35}
\]

Form the figure, we can see that the bit transmission rate of the proposed scheme is much higher than the PPM scheme [3] when \(M\) increases and it goes to the constant value, \((N_s N_h T_m)^{-1} \approx 40 \text{ Mbps}\) while the bit transmission rate of the PPM scheme [3] goes to zero.

Next, we consider the BER performance of the proposed scheme. By using Eqn. (33), we investigate the BER performance of the proposed scheme. For comparison, the BER performance of the PPM scheme [3] is included. The BER performance is shown in Fig. 2 when \(M = 3, 5, 7\). For this, we let \(N_s = 8\) and \(E_b/N_0\) lies in the range \([-12 \text{ dB}, -2 \text{ dB}]\). By letting the bit transmission rate to be same for both of two schemes, the proposed scheme shows better BER performance than the PPM scheme as \(M\) increases. It is because the proposed scheme has a \(\frac{1}{(M+1)}\) times more spreading gain than the PPM scheme.

VI. CONCLUSION

In this paper, we proposed an UWB time-hopping multiple-access (TH-MA) scheme using the multi-code based PPM. The BER performance and bit transmission rate of the proposed scheme is analyzed. Through numerical results, we can see that the proposed scheme can achieve the much higher transmission rate than the conventional PPM scheme [3] and due to this merit of the very high transmission rate, it can achieve better BER performance than the conventional PPM scheme [3] in the same bit transmission rate.

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