LETTER

Dimension-Reduced MMSE Receiver for DS-CDMA Systems over Multipath Channels

Kuk-Jin SONG† and Dong-Jo PARK†, Nonmembers

SUMMARY

A new dimension-reduced interference suppression scheme is proposed for DS-CDMA systems over multipath channels. The proposed receiver resolves the problems of interference and multipath effects without needing to estimate the channel and training sequences. The minimum mean squared error (MMSE) criterion is used to obtain an algorithm to cancel the interference of each path. The MMSE filter is composed of two stages based on multipath effects. The proposed receiver has low complexity without great degradation of performance compared with the full dimension MMSE receiver with known channel information. Simulation results show that the proposed receiver converges to the optimal value rapidly because of its reduced dimension.

KEY WORDS: DS-CDMA, near-far problem, multipath, interference suppression

1. Introduction

The DS-CDMA systems in wireless and mobile communications have great advantages such as asynchronous multiple access and robustness to frequency selective fading. However, the performance of receivers is degraded by various drawbacks such as the near-far problem due to the multiple access interference (MAI), multipath effects and channel noise. The near-far problem is caused by non-equal power of each user’s signal of the system, especially when other users’ power is greater than the interested user’s. The multipath effect results in inter-symbol interference (ISI). Many kinds of adaptive algorithms have been proposed to overcome these drawbacks with simple implementations.

Among the adaptive algorithms, the minimum mean squared error (MMSE) receiver [1] effectively removes the near-far problem although it requires the estimation of the channel. Subsequently, the blind adaptive receiver [2] is introduced to make up for the weak points in the MMSE receiver which needs training sequences known to the receiver for initial adaptation. The conventional RAKE receiver is the popular receiver developed to eliminate the multipath effects. However, the RAKE receiver cannot suppress interference by itself.

The dimension of the conventional adaptive receiver in the DS-CDMA systems is equal to the processing gain. Therefore, if the system has very long spreading codes, the computational complexity may be a burden to the implementation of the receiver and the receiver may slowly converge due to the large dimension. In this paper, we focus on the reduction of the dimension of the receiver in the presence of the interference and multipath effects. The previously proposed dimension-reduced receiver [3] using partial decorrelation for reducing the dimension suppresses interference effectively over non-multipath channels.

However, the receiver requires the delay of the interested user’s signal and training sequences. The dimension-reduced receiver proposed in [4] is designed considering multipath effects. Most of the proposed receivers [3]–[5] utilized the partial decorrelation to reduce the dimension. We propose partial summation as a reduction scheme for the simple implementation of the receiver. The scheme makes it possible to use the MMSE criterion which is not applicable to the receiver in [4].

The proposed receiver is similar to the receiver in [4], but it has a simpler structure and uses the MMSE criterion for better performance. The functions of the proposed receiver are divided into three parts. In the front part, the dimension of the received data are reduced for the dimension-reduced suppression filter. The next two stages are composed of the stages to suppress interference and to eliminate the ISI using maximum combining.

This paper is organized as follows: In Sect. 2, we describe the asynchronous DS-CDMA system model and problem formulation. In Sect. 3, the proposed receiver is designed. The simulation results of the proposed receiver are compared with results of other receivers in Sect. 4. Finally, we will draw conclusions of this paper in Sect. 5.

2. Problem Formulation

Assume K-user asynchronous DS-CDMA systems using binary phase shift keying (BPSK) modulation where each user has J resolvable paths. The kth user’s bit symbol, $b_k(m)$ to be transmitted at the nth time is spread by a spreading waveform, $s_k(t)$ uniquely assigned to the specific user as
where $T_s$ is the bit duration. The spreading waveform $s_k(t)$ is given by

$$s_k(t) = \sum_{n=1}^{N} c_k(n)p(t - nT_c)$$  

where $T_c = T_s/N$ is the chip duration, $c_k(n)$ is the spreading sequence of the $k$th user, $p(t)$ is the chip waveform and $N$ is the processing gain. Then, the received baseband signal at the $m$th bit symbol time can be expressed as

$$x_k(t) = \sum_{m} b_k(m)s_k(t - mT_s)$$ \hspace{1cm} (1)$$

where $\sigma_n^2$ is the mean additive white Gaussian noise with the variance $\sigma_n^2$.

The received vector after sampling is given by

$$r(t) = \sum_{j=1}^{J} \sum_{k=1}^{K} \alpha_{k,j}x_k(t - \tau_{k,j}) + n(t)$$ \hspace{1cm} (3)$$

where $\alpha_{k,j}$ and $\tau_{k,j}$ are the $k$th user’s path gain and delay of the $j$th path, respectively. $n(t)$ is the zero mean additive white Gaussian noise with the variance $\sigma_n^2$.

Then, the received vector after sampling is given by

$$r(m) = [r(0), r(1), \ldots, r(N + L - 1)]^T$$

$$= \sum_{j=1}^{J} \sum_{k=1}^{K} \alpha_{k,j}c_{\tau_{k,j}}b_k(m) + \mathbf{i}(m) + \mathbf{n}(m)$$

$$= \sum_{k=1}^{K} \bar{c}_k b_k(m) + \mathbf{i}(m) + \mathbf{n}(m)$$ \hspace{1cm} (5)$$

where $\mathbf{r}(m)$ is an $(N + L - 1) \times 1$ vector, $\mathbf{i}(m)$ includes all users’ signals contributed by $(m-1)$th and $(m+1)$th bit symbols due to time delays, $\mathbf{n}(m)$ is the noise vector, and $c_{\tau_{k,j}}$ is one of the columns of the $(N + L - 1) \times L$ matrix assuming $\tau_{k,j}$ is the integer multiple of $T_c$:

$$C_k = \begin{bmatrix}
    c_k(0) & 0 & \cdots & 0 \\
    \vdots & c_k(0) & \cdots & \vdots \\
    c_k(N-1) & \vdots & \ddots & 0 \\
    0 & c_1(N-1) & \cdots & c_k(0) \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \cdots & c_k(N-1)
\end{bmatrix}.$$ \hspace{1cm} (6)$$

$\bar{c}_k$ in (5) is the effective spreading code of the $k$th user can be defined by

$$\bar{c}_k = \sum_{j=1}^{J} \alpha_{k,j} c_{\tau_{k,j}} = C_k h_k$$ \hspace{1cm} (7)$$

where $h_k$ is the $k$th user’s channel response as

$$h_k = [\alpha_{k,0T_c}, \alpha_{k,T_c}, \ldots, \alpha_{k,(L-1)T_c}]^T.$$ \hspace{1cm} (8)$$

In (8), $\alpha_{k,lT_c}$ is the sum of the path gains of the $k$th user when $\tau_{k,l} = lT_c$ where $l$ is an integer.

Without loss of generality, we consider the signal from the first user as the desired signal and the signals from all other users as interfering signals throughout the paper. Then, we can rewrite the received vector as follows:

$$\mathbf{r}(m) = \mathbf{c}_1 b_1(m) + \mathbf{u}(m)$$ \hspace{1cm} (9)$$

where $\mathbf{u}(m)$ includes interfering signals from other users, interested user’s ISI and the additive noise. Now the problem is to detect the bit symbol $b_1(m)$ from the received signal (9).

3. Receiver Design

The objective of the proposed receiver design is to develop a low complexity (reduced dimension) receiver without great loss of performance. The structure of the proposed receiver is depicted in Fig. 1. It is similar to the D-RAKE receiver in [6] except that partially adaptive methods are applied to reduce the dimension of the receiver and the MMSE criterion is used. Partial adaptivity can be achieved by either working with a smaller data vector or with a smaller blocking matrix [5].

The proposed receiver has three stages. In the stage 1 the dimension of the received data is reduced. The stage 2 comprises a set of adaptive weight vectors, $\{\mathbf{g}^l\}$, which suppresses the interference in the $l$-chip delayed signal. The stage 3 estimates the channel response using maximum ratio combining.

3.1 Reduction of Received Data

The received data is transformed into the dimension-reduced data vector. Let $\mathbf{P}$ be the $(N+L-1) \times (N_n + L-1)$ matrix and the dimension reduced vector $\mathbf{r}'$ is calculated as

$$\mathbf{r}' = \mathbf{P} \mathbf{r}.$$
1) transform matrix which transforms an \((N + L - 1) \times 1\) vector into an \((N_n + L - 1) \times 1\) vector:

\[
\mathbf{r}^l(m) = \mathbf{P}^T \mathbf{r}(m)
\]

(10)

where the dimension of \(\mathbf{r}^l(m)\) is \(N_n + L - 1\) and \(N_n < N\).

Constructing the transform matrix \(\mathbf{P}\) may affect the performance of receivers as well as the dimension \(N_n + L - 1\). The proposed reduction scheme is to make a new received vector from the outputs of the partial summation of the received data. Then the transform matrix using the partial summation can be

\[
\mathbf{P} = \begin{bmatrix} 1^{(N \Delta)} & 1^{(N + L - 1)} & \cdots & 1^{(N + L - 1)} \end{bmatrix}
\]

(11)

where \(1^{(n; m)}\) is an \((N + L - 1) \times 1\) vector with its \(n\)th to \(m\)th entries are 1’s and zeroes elsewhere, and \(\Delta = (N + L - 1)/(N_n + L - 1)\). It is easy to modify the reduced data vector using overlapping. The transformation matrix in (11) partially sums the neighboring received data. The partial decorrelation of the previous receivers may outperform the partial summation because the partial decorrelation itself suppresses interference to some extent. However, the scheme using the partial decorrelation requires \(L\) transformation matrices [4] according to each finger. The proposed scheme using the partial summation needs only one transformation matrix. Besides, the partial summation includes the information of the reduced data and the MMSE criterion can be applicable. Other dimension reducing schemes described in [3], [5] require known timing delay or no multipath to employ the MMSE criterion.

3.2 Interference Suppression and Maximum Ratio Combining

In this subsection the MMSE is applied to the dimension-reduced received data \(\mathbf{r}^l(m)\) in (10). The MMSE cost function is

\[
J_{\text{MMSE}} = E\left\{ (b_1(m) - \mathbf{w}_l^T \mathbf{P}^T \mathbf{r}(m))^2 \right\}.
\]

(12)

The optimal solution to the cost function (12) is

\[
\mathbf{w}_l_{-\text{MMSE}} = (\mathbf{P}^T \mathbf{R}_l \mathbf{P})^{-1} \mathbf{P}^T \mathbf{c}_l
\]

\[
= (\mathbf{P}^T \mathbf{R}_l \mathbf{P})^{-1} \mathbf{P}^T \mathbf{C}_l \mathbf{h}_1
\]

\[
= \sum_{l=1}^{L} h_1(l) (\mathbf{P}^T \mathbf{R}_l \mathbf{P})^{-1} \mathbf{P}^T \mathbf{C}_l 1_l
\]

(13)

where \(\mathbf{R}_l = E\{\mathbf{r}(m)\mathbf{r}^T(m)\}\) is the autocorrelation matrix of the received vector and \(1_l\) is a vector with all elements 0’s except 1 at the \(l\)th position.

The channel information, \(h(l), \ l = 1, \ldots, L\) and the autocorrelation matrix \(\mathbf{R}_l\) must be estimated to obtain the solution in (13). We construct the weight vector of (13) through two stages. First, we form the weight vector of each finger at the second stage as

\[
g_l^1 = (\mathbf{P}^T \mathbf{R}_l \mathbf{P})^{-1} \mathbf{P}^T \mathbf{C}_l 1_l
\]

(14)

without the channel estimation. The channel coefficients can be estimated at the third stage using the outputs of the second stage.

Let \(z^l\) be the output of the \(l\)th finger using the MMSE criterion, then the output can be given by

\[
z^l(m) = g_l^1 \mathbf{P}^T \mathbf{r}(m)
\]

\[
= 1_l^T (\mathbf{C}_p^T \mathbf{R}_l^{-1} \mathbf{C}_p) \mathbf{h}_1 b_1(m) + u_{z^l}(m)
\]

(15)

where \(\mathbf{C}_p = \mathbf{P}^T \mathbf{C}_1, \mathbf{R}_p = \mathbf{P}^T \mathbf{R}_l \mathbf{P}\), and \(u_{z^l}(m)\) is the effective noise and interference after filtering with (14).

Stacking of the \(L\) outputs leads to the following matrix-vector form:

\[
\mathbf{z}(m) = [z^1(m) \ z^2(m) \ \cdots \ z^L(m)]^T
\]

\[
= (\mathbf{C}_p^T \mathbf{R}_p^{-1} \mathbf{C}_p) \mathbf{h}_1 b_1(m) + \mathbf{u}_z(m)
\]

(16)

where \(\mathbf{u}_z(m) = [u_{z^1}(m) \ u_{z^2}(m) \ \cdots \ u_{z^L}(m)]^T\).

If each finger \(g_l^1\) can suppress interference sufficiently, the autocorrelation matrix of \(\mathbf{z}(m)\) can be

\[
\mathbf{R}_z = E\{\mathbf{z}(m)\mathbf{z}^T(m)\}
\]

\[
= (\mathbf{C}_p^T \mathbf{R}_p^{-1} \mathbf{C}_p) \mathbf{h}_1 \mathbf{h}_1^T (\mathbf{C}_p^T \mathbf{R}_p^{-1} \mathbf{C}_p) + \mathbf{R}_{u_z}
\]

\[
\approx (\mathbf{C}_p^T \mathbf{R}_p^{-1} \mathbf{C}_p) \mathbf{h}_1 \mathbf{h}_1^T (\mathbf{C}_p^T \mathbf{R}_p^{-1} \mathbf{C}_p)
\]

(17)

where \(\mathbf{R}_{u_z}\) is the autocorrelation matrix of \(\mathbf{u}_z\). Because the interference in the received data is suppressed through the second stage filtering, \(\mathbf{R}_{u_z}\) can be estimated as following:

\[
\mathbf{R}_{u_z} = E\{\mathbf{u}_z(m)\mathbf{u}_z^T(m)\}
\]

\[
= \sigma_n^2 \mathbf{G}^T \mathbf{P} \mathbf{G}
\]

\[
= \sigma_n^2 \mathbf{C}_p (\mathbf{R}_p^2)^{-1} \mathbf{C}_p
\]

(18)

where \(\mathbf{G} = [\ g_1^1 \ g_2^1 \ \cdots \ g_L^1 \ ]\). The unknown parameter \(\sigma_n^2\) has no effect on finding the third stage’s weight vector.

The weight vector, \(\mathbf{w}\) for the maximum ratio combining can be determined as the solution of the following cost function:

\[
\max_{\mathbf{w}} \frac{E\{||\mathbf{w}^T \mathbf{z}(m)||^2\}}{E\{||\mathbf{w}^T \mathbf{u}(m)||^2\}} = \frac{\mathbf{w}^T \mathbf{R}_z \mathbf{w}}{\mathbf{w}^T \mathbf{R}_{u_z} \mathbf{w}}.
\]

(19)

The solution for (19) is the principle generalized eigenvector of the matrix pair \(\{\mathbf{R}_z, \mathbf{R}_{u_z}\}\) [7]. Using the estimated \(\mathbf{R}_z\) and \(\mathbf{R}_{u_z}\), we can obtain \(\mathbf{w}\) maximizing the cost function (19) and then \(\mathbf{h}_1 \approx \eta \mathbf{w}\) where \(\eta\) is a scalar value. \(\mathbf{h}_1\) can be estimated by finding the eigenvector which has the maximum eigenvalue of the following matrix comprised of known matrices, as well:

\[
(\mathbf{C}_p^T \mathbf{R}_p^{-1} \mathbf{C}_p)^{-1} \left[ \mathbf{R}_z - \sigma_n^2 \mathbf{C}_p^T (\mathbf{R}_p)^{-1} \mathbf{C}_p \right] (\mathbf{C}_p^T \mathbf{R}_p^{-1} \mathbf{C}_p)^{-1}.
\]

(20)

4. Simulation Results

The proposed MMSE based receiver is evaluated
through numerical examples. As a performance measure, we use the output signal-to-interference-noise ratio (SINR) of the receivers. The output SINR of the proposed receiver can be defined as follows:

$$\text{SINR}_{\text{reduced}} = \frac{||w_{\text{MMSE}}^T P_c \hat{c}_1||^2}{||w_{\text{MMSE}}^T P_c R_c P_c w_{\text{MMSE}}||^2}.$$  \hspace{1cm} (21)

The input SNR is defined to be $E\{ \hat{b}_1(m)^2/\sigma_n^2 \}$ and the near-far-ratio (NFR) is the ratio of the signal power to the MAI power before despreading. The path gains $\alpha_{k,j}$’s are assumed independent, identically distributed unit variance Gaussian random variables, the path delays $\tau_{k,j}$’s are assumed uniform over $[0,3T_c]$, and the number of resolvable paths is $J = 4$ for all users.

All simulations involved $K$ CDMA signals spread by the randomly generated spreading code of length $N = 40$ and BPSK modulation schemes are used. The number of fingers is $L = 4$ and the input SNR is 10 dB, that is, high SNR. A total 50 Monte-Carlo runs are executed to obtain the resulting statistics.

For comparison, we include the results of the proposed receiver, the full-dimension MMSE receiver and the conventional RAKE receiver. The weight vector of the MMSE receiver is $\hat{w}_{\text{MMSE}} = R_c^{-1} \hat{c}_1$, and that of the conventional RAKE receiver is $\hat{w}_{\text{RAKE}} = \hat{c}_1$. In practical implementations the autocorrelation matrix must be estimated. For simulations in this section the autocorrelation matrix is estimated simply as follows:

$$\hat{R}_c = \frac{1}{M_s} \sum_{m=1}^{M_s} r(m) r^T(m).$$  \hspace{1cm} (22)

Throughout all simulations, it is assumed that the effective spreading code, $\hat{c}_1$ is exactly known to the MMSE receiver and the conventional RAKE receiver. The proposed receiver does not know the channel response.

The resulting output SINR’s versus $M_s$ are plotted in Fig. 2 with $N_n = 10$, NFR = 0 dB for $K = 5$, $N_n = 10$.

In Fig. 2, ‘Proposed’ represents the output SINR of the proposed receiver. As expected, the output SINR increases as $M_s$ increases for all receivers except the RAKE receiver. The MMSE receiver whose dimension is $(N + L - 1) \times 1$ converges more slowly than the proposed receiver whose dimension is $(N_n + L - 1) < (N + L - 1)$. The output SINR of the proposed receiver is better than that of the MMSE receiver until $M_s = 1500$.

Next, the robustness against the change of interference powers called near-far resistance, is evaluated in Fig. 3 for $K = 5$ and $N_n = 10$.

Because $M_s = 500$ in this simulation, the proposed receiver outperforms the MMSE receiver. As observed from Fig. 3, the proposed receiver and the MMSE receiver achieve excellent near-far resistance by successful rejection of the MAI.

The final simulation results comparing the partial summation and partial decorrelation is shown in Fig. 4. For simplicity, we assume that the delay of the desired signal is known to receivers and the channel has no multipath effect. Because the synchronous CDMA system is considered in this simulation, both proposed partial summation and partial decorrelation schemes require one transformation matrix. Therefore, they have the same computational complexity if they have equal dimensions.

From Fig. 4, the method of partial decorrelation outperforms that of the partial summation. However, the difference in the output SINR between two schemes becomes smaller as $N_n$ becomes larger because the degree of freedom is increased. The reason of poorer performance of the partial summation scheme when $N_n = 5$ is that the degree of freedom is equal to the number of users, $K = 5$. The figure shows that the reduced dimension has effect on the performance of the receiver.

As seen in Fig. 4, the partial decorrelation scheme has lower computational complexity than the partial summation scheme under the condition that almost the same performance is obtained because it has a lower dimension than the partial summation scheme. In the
asynchronous systems covered in the whole paper, the receiver using the partial decorrelation scheme with dimension $N_n = 5$ may outperform the receiver using the partial summation scheme with dimension $N_n = 10$ or $N_n = 20$. However, the computational complexity of the receiver using the partial decorrelation scheme grows according to the number of resolvable paths because it requires transformation matrices as many as the number of fingers. Each finger’s weight vector of the partial decorrelation scheme in [4] to suppress interference requires more calculations than that of the partial summation scheme described in (14). Besides, the method in [4] requires signal blocking matrices used in each finger. The partial summation scheme requires only one transformation matrix in the asynchronous system as well. The one transformation matrix makes it easy to implement the second stage weight vectors as (14). The $l$th finger’s weight vector of the partial summation scheme is the $l$th column of the matrix $(P^T R P)^{-1} P^T C_1$. Therefore, the partial summation scheme whose dimension is greater than that of the partial decorrelation scheme may have lower computational complexity than the partial decorrelation scheme under the condition that almost the same performance is obtained in the multipath environments. Of course, if both schemes have the same dimension, the proposed partial summation scheme has lower computational complexity than the method in [4].

5. Conclusions

An interference suppression scheme with a reduced degree of freedom is suggested for DS-CDMA communications over multipath channels. The proposed receiver reduces system complexity and computational burden. In addition, the proposed receiver converges to the optimum faster than the full dimension receiver and we clarified the properties through computer simulations. From simulation results, the near-far resistance of the proposed receivers is also proven. For further works, more efficient schemes to reduce dimension of the systems and the optimal selection of the reduced dimension will be studied. A reduction scheme applicable to systems with aperiodic spreading codes should be examined.

References