NEW PERFORMANCE FUNCTION AND VARIABLE STEP SIZE LMS ALGORITHM DERIVED BY KARNI AND ZENG

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The variable step size LMS algorithm (Karni-Zeng) can be derived from a new performance function which has an additional exponential term to that of the conventional LMS algorithm. Its fast convergence and low misadjustment error properties can be interpreted and analyzed through the new performance function. The algorithm uses a large step size when the gradient on the surface of the performance function is large and a small step size when the gradient is small.

Introduction: The LMS (least mean square) algorithm was developed by Widrow in 1959 and originally applied to a neural network known as Adaline. Since then the LMS algorithm has been successfully applied to a number of practical applications such as adaptive filters in telecommunication, radio and audio applications due to its robustness in nonstationary environments and its simplicity in computation. However, the main drawback of the LMS algorithm is the tradeoff relationship of the convergence rate and the residual misadjustment errors after convergence.

To overcome this difficulty, many approaches have been studied. Most of them are the methods of controlling the step size (or adaptive feedback gain) \( \mu \) according to a number of available pieces of information. In this Letter we derive a new performance function which has an additional exponential term to that of the conventional LMS algorithm (Karni-Zeng) in a modified form such that the algorithm converges around the optimal weight vector, the norm of the gradient on the surface of the performance function is greatly reduced, and \( \mu(n) \) becomes very small. Hence the algorithm is in the misadjustment error minimizing state.

These properties can be seen from the performance functions on Fig. 1 and their gradients in Fig. 2. From the effect of the additional exponential term, the curvature of the new performance function around its optimal weight is flatter than that of the original LMS algorithm in Fig. 1. The gradients in the neighbourhood of the optimal weight vector are also reduced as shown in Fig. 2. Suppose that \( |a(n)X(n)| \) is large, then the gradient on the surface of the performance function is large and the algorithm converges very quickly. When the filter weight vector approaches the optimal value, \( |a(n)X(n)| \)
becomes very small. Hence the new performance function becomes wider for small input data \(X(n)\) and the curvature of the performance function in the neighbourhood of the optimal values is flat. Consequently the gradient on the surface of the performance function is very small and the misadjustment error is significantly reduced.

The fast convergence and small misadjustment error properties can be confirmed from computer simulations in Fig. 3 as expected even for the nonstationary environments.

![Fig. 3 Comparison of convergence properties](image)

The filter weight vector in computer simulations is abruptly switched to another value on the 200th iteration. Design parameters used for simulations are as follows: \(N = 5\), \(\mu = 0.15\), and \(\sigma = 5\), which are dependent on the statistics of the input data vector \(X(n)\) that is generated from the output sequence of a filter with a white Gaussian noise input signal.

The convergence rate depends on the data vector \(X(n)\) so that if input power \(|X(n)|^2\) is high, the speed of convergence is high. In contrast, for low input power, the gradient \(e(n)X(n)\) is small for the same error level, and the convergence rate is reduced. When the optimal filter weight vector is located around the eigenvector corresponding to the smallest eigenvalue when the filter weight vector starts from the initial point, the convergence speed is very low. On the other hand if the optimal vector is along the direction of the eigenvector of the largest eigenvalue, the rate of convergence is highest. These differences of convergence rate are observed in the computer simulation in Fig. 3. The algorithm often encounters slow convergence in the environment of weak data signals or when the initial condition happens to be in the direction of the eigenvector with a small eigenvalue as shown in the computer simulations.

Conclusions: The variable step size LMS algorithm is derived from a new performance function which has an additional exponential term to that of the LMS algorithm. From these results the introduction of the variable step size LMS algorithm is justified and its fast convergence and low misadjustment error properties can be easily interpreted and analysed.

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References

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